A gyrokinetic model for the tokamak periphery

R. Jorge[1,2,*], B. Frei[1], P. Ricci[1], N. F. Loureiro[3], A. Baillod[1], S. Gamba[1], L. Perrone[1], K. Vyacheslav[1]

[1]École Polytechnique Fédérale de Lausanne (EPFL), Swiss Plasma Center (SPC), CH-1015 Lausanne, Switzerland
[2]Instituto de Plasmas e Fusão Nuclear, Instituto Superior Técnico, Universidade de Lisboa, 1049-001 Lisboa, Portugal
[3]Plasma Science and Fusion Center, Massachusetts Institute of Technology, 02139 Cambridge, Massachusetts, USA

*rogerio.jorge@epfl.ch

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

SWISS PLASMA CENTER

ipfn INSTITUTO DE PLASMAS E FUSÃO NUCLEAR

TOKAMAK PERIPHERY

Small-Scale Fluctuations

$$k_\perp \rho_i \sim 1 \qquad \frac{e\phi}{T_e} \ll 1$$

Large-Scale Fluctuations

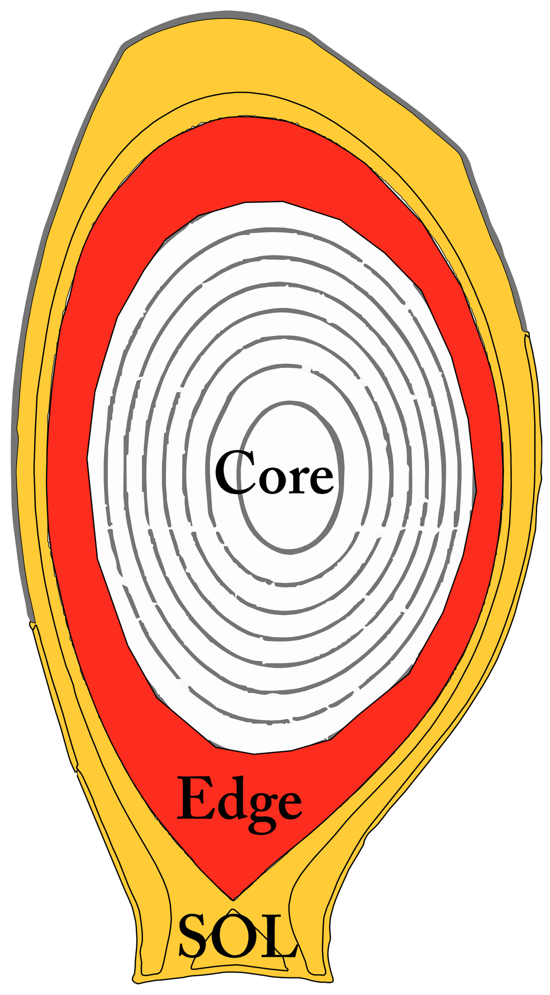
$$k_\perp \rho_i \ll 1 \qquad \frac{e\phi}{T_e} \sim 1$$

Large to Small Collisionalities

$$\lambda_{mfp} \gtrless L_\parallel$$

Low Frequencies

$$\omega \ll \Omega_i$$



MODELLING ASSUMPTIONS AND APPROACH

Ordering

$$\epsilon = k_\perp \rho_i \frac{e\phi}{T_e} \sim \frac{k_\parallel}{k_\perp} \sim \frac{\phi_1}{\phi_0} \ll 1$$

Arbitrary Collisionality

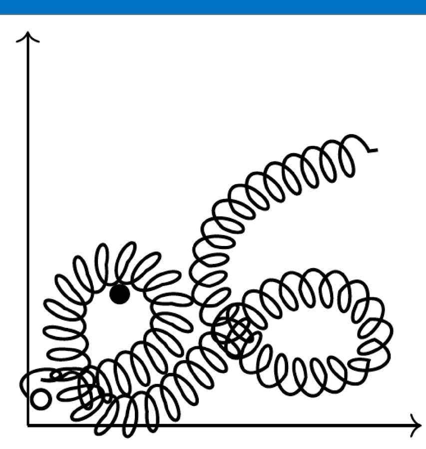
$$\frac{\lambda_{mfp}}{L_\parallel} \gtrless 1$$

Single Particle Dynamics → Gyrokinetic Theory → Moment Hierarchy

Retain

- Both large scale and small scale fluctuations
- Electromagnetic and full-F effects
- Full Coulomb collisions
- Numerical efficiency

SINGLE PARTICLE DYNAMICS



Particle Lagrangian $L(\mathbf{x}, \mathbf{v})$

$$L = q\mathbf{A}\cdot\dot{\mathbf{x}} - q\phi - \frac{mv^2}{2}$$

$$k_\perp \rho_i \ll 1 \qquad \frac{e\phi_0}{T_e} \sim 1$$

Split parallel and perpendicular velocities

$$\Gamma = q\mathbf{A}^*\cdot\dot{\mathbf{R}} - q\phi_0^* - \frac{mv_\parallel^2}{2} + \mu\frac{m\dot\theta}{q}$$

$$k_\perp \rho_i \sim 1 \qquad \frac{e\phi_1}{T_e} \ll 1$$

Introduce small scale fluctuations

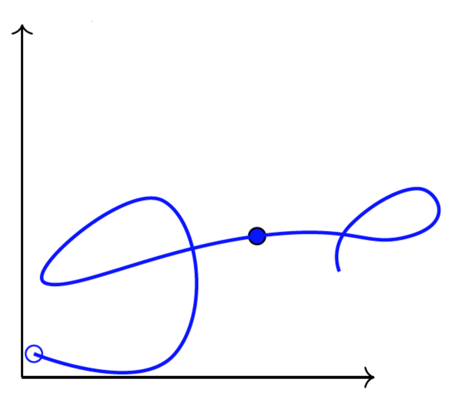
$$\overline{\Gamma} = \Gamma + \Gamma_1 = \Gamma - q\langle\Phi\rangle + \frac{q^2}{2m}\left\langle A_{\parallel 1}^2\right\rangle - \frac{q^3}{2m\Omega}\frac{\partial}{\partial\mu}\left(\langle\Phi^2\rangle - \langle\Phi\rangle^2\right)$$

Equations of Motion

$$\mathbf{U} = v_\parallel \mathbf{b} + \mathbf{v}_{\mathbf{E}\times\mathbf{B}}$$

$$B_\parallel^* = B\left(1 + v_\parallel\frac{\mathbf{b}\cdot\nabla\times\mathbf{b}}{\Omega} + \frac{\mathbf{b}\cdot\nabla\times\mathbf{v}_{\mathbf{E}\times\mathbf{B}}}{\Omega}\right)$$

$$\Phi = \phi_1 - v_\parallel A_{\parallel 1}$$



Gyrokinetic Lagrangian $\overline{\Gamma}(\mathbf{R}, v_\parallel, \mu)$

$$q\mathbf{B}^*\times\dot{\mathbf{R}} + mv_\parallel\mathbf{b} = -\nabla\left(q\phi_0 + q\Gamma_1 + m\frac{v_\parallel^2 + v_{\mathbf{E}\times\mathbf{B}}}{2}\right), \quad \dot\mu = 0$$

Gyrokinetic Equation

$$\frac{\partial F}{\partial t} + \dot{\mathbf{R}}\cdot\nabla F + \dot{v}_\parallel\frac{\partial F}{\partial v_\parallel} = \langle C(F)\rangle$$

+ Maxwell's equations using a variational principle

GYROKINETIC MOMENT HIERARCHY

$$\int (\text{GK Eq.})\, H_p L_j dv_\parallel d\mu \qquad F = F_M \sum_{p,j} N^{pj}(\mathbf{R}) H_p(v_\parallel) L_j(\mu)$$

Hermite Polynomials, Laguerre Polynomials

$$\boxed{\frac{\partial N^{pj}}{\partial t}} + \boxed{\nabla\cdot\dot{\mathbf{R}}^{pj}} - \boxed{\frac{\sqrt{2p}}{v_{th}}\dot{v}_\parallel^{p-1j}} + \boxed{\mathcal{F}^{pj}} = \boxed{C^{pj}}$$

Time Evolution; Spatial evolution of Moments + Fields; Forces included at p>0; Fluid Operator (density, velocity, temperature); Collisions

3-D moment-hierarchy equation for the evolution of the coefficients $N^{pj}(\mathbf{R})$

Advantages of a moment-hierarchy equation

- Set of fluid-like equations with tunable computational cost
- Number of moments chosen according to the desired level of accuracy (function of collisionality)
- Reduce to the fluid model at high collisionality

$$\mathbf{R}^{pj} = \left(1 + \frac{\mathbf{b}}{\Omega^*}\times\frac{d}{dt}\right)\left(u_\parallel\mathbf{b} + \mathbf{v}_{\mathbf{E}\times\mathbf{B}}\right)N^{pj} + \text{curvature} + \text{grad-B}$$

$$+ v_{th}\mathbf{b}\left(\sqrt{\frac{p+1}{2}}N^{p+1j} + \sqrt{\frac{p}{2}}N^{p-1j}\right) + \frac{\mathbf{b}}{B}\times\int H_p L_j \nabla\langle\Gamma_1\rangle dv_\parallel d\mu$$

Moments of gyroaverages computed using

$$\langle\Gamma_1\rangle = J_0(k_\perp\rho)\Gamma_1 = \sum_{n=0}^{\infty}\frac{L_n(\mu)}{n!}\left(\frac{k_\perp\rho_{th}}{2}\right)^{2n} e^{-\left(\frac{k_\perp\rho_{th}}{2}\right)^2}\Gamma_1$$

COLLISION OPERATOR

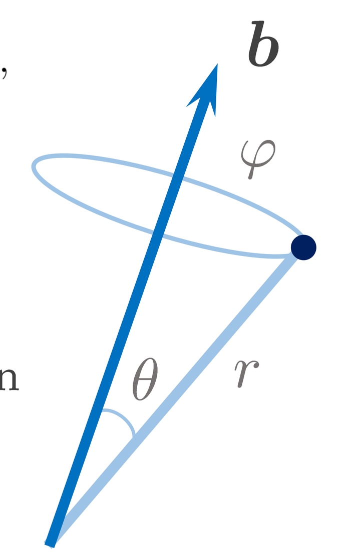
$$C(F) = \sum_b \frac{\nu_{ab}v_{tha}^3}{n_b}\sum_{i,j}\frac{\partial}{\partial v_i}\left[\frac{\partial^2 G_b}{\partial v_i\partial v_j}\frac{\partial f_a}{\partial v_j} - \frac{m_a}{m_b}\frac{\partial H_b}{\partial v_i}f_a\right] \qquad H_b = 2\int\frac{f_b(\mathbf{v}')}{|\mathbf{v}-\mathbf{v}'|}d\mathbf{v}'$$

Multipole Expansion of the Rosenbluth Potentials

$$\frac{1}{|\mathbf{v}-\mathbf{v}'|} = \begin{cases}\sum_l \frac{(-\mathbf{v}')^l}{l!}\cdot\partial_{\mathbf{v}}^l\left(\frac{1}{v}\right), & v' \le v,\\ \sum_l \frac{(-\mathbf{v})^l}{l!}\cdot\partial_{\mathbf{v}'}^l\left(\frac{1}{v'}\right), & v < v'\end{cases}$$

Identity between spherical harmonic tensor and scalar

$$v^{l+1}\left(\frac{\partial}{\partial\mathbf{v}}\right)^l\frac{1}{v} = C\sum_{m=-l}^{l}Y_{lm}(\theta,\varphi)\mathbf{e}^{lm}$$



Expanding $f \sim Y_{lm}(\theta,\varphi)L_k^{l+1/2}(v^2)$ and using a basis transformation between spherical harmonics and Hermite-Laguerre polynomials

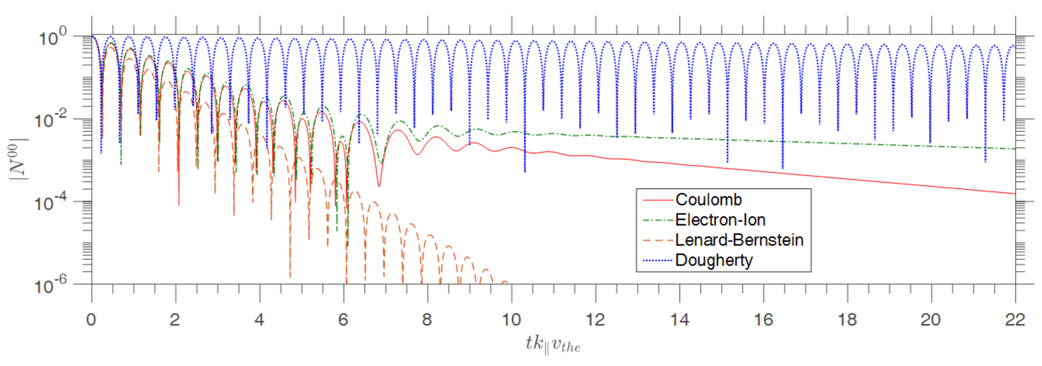
$$v^l Y_{lm}(\theta,0)L_k^{l+1/2}(v^2) = \sum_{p=0}^{l+2k}\sum_{j=0}^{k+l/2}T_{lk}^{pjm}H_p(v_\parallel)L_j(\mu)$$

Hermite-Laguerre formulation of the Coulomb collision operator readily found

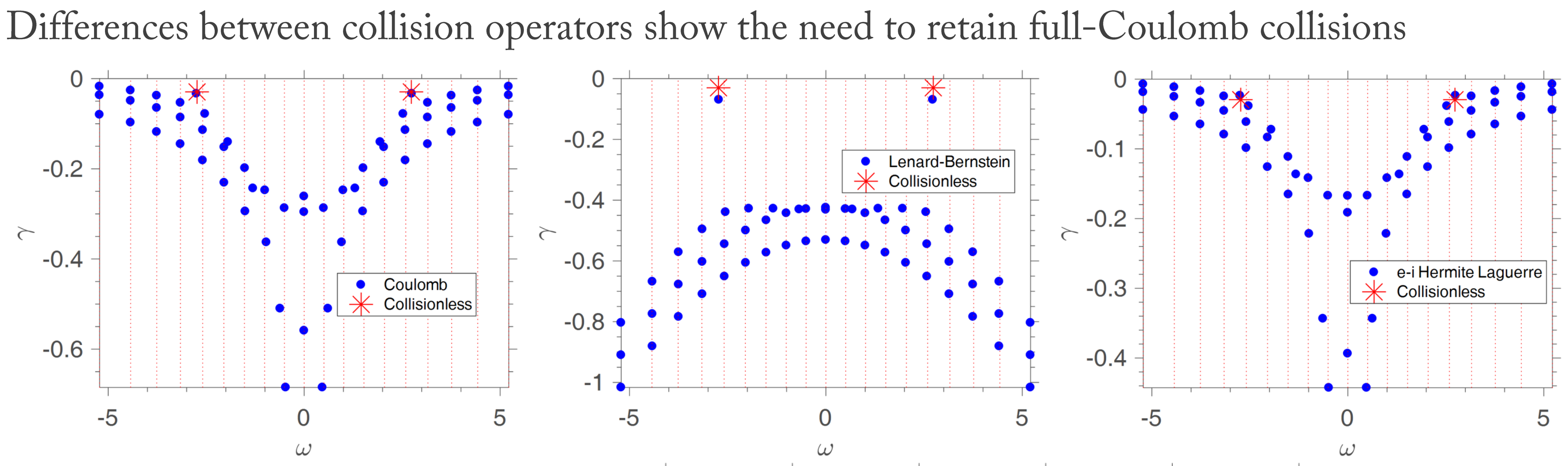
NUMERICAL RESULTS

Electron Plasma Waves

The entropy zero-frequency mode, characteristic of Coulomb collisions, emerges as the long term behavior

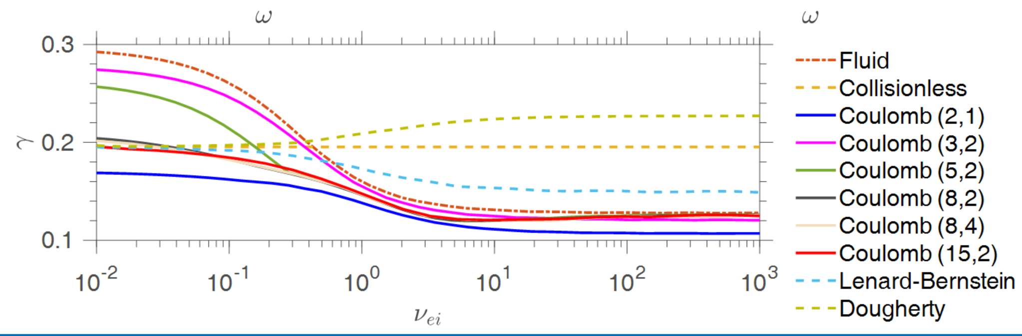


Differences between collision operators show the need to retain full-Coulomb collisions



Drift Wave Instability

Comparison between different collision operators at arbitrary collisionalities



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